Pre TPS

Dimensions and Units Tutorial
Solutions

1. a. MLT^{-2}; b. MLT^{-2}; c. L; d. T; e. L^3; f. ML^2T^{-2}

2. $M^{-1}L^3T^{-2}$ a. metric – nt.m^2/kg^2; b. imperial – lbs.ft^2/$slug^2$ (note that these are not exhaustive)

3. a. yes; b. no; c. yes; d. yes; e. no; f. yes.

4. 42,320 kms

5. ΔP = 52.76 lbs/sq.ft. = 0.37 lbs/sq.in. = 0.025 atmospheres

Name: _________________

Algebra Tutorial Solutions

1. Combine:
   a. $2x + (3x - 4y)$ — $5x - 4y$

   b. $4x^2 + 5x - (3x - 7) + (-2x^2 + 3)$ — $2x^2 + 2x + 10$

   c. $[(x + 2y) - (x + 3y)] - [(2x + 3y) - (-4x + 5t)]$

   $-6x - 4y + 5t$

2. Add:
   a. $x^2 + 2x - 1 + 3x - 4 + 2x^2 + 5$ — $3x^2 + 5x$

   b. $7x + 3y^3 - 4xy$, $3x - 2y^3 + 7xy$, $2xy - 5x - 6y^3$

   $5x - 5y^3 + 5xy$

   c. $\theta + \alpha + 2\alpha - \theta + 3\theta + 4\alpha$ — $7\alpha + 3\theta$

3. Subtract:
   a. $2x^2 - 3xy + 5y^2$ from $10x^2 - 2yx - 3y^2$

   $8x^2 + xy - 8y^2$

4. Remove brackets and simplify:

a. $2(x^2 - 4x)$ — $2x^2 - 8x$

b. $-a(2a + 3b)$ — $-2a^2 - 3ab$

c. $2x[-4(3 + 2y) + (x + y + 1)]$ — $2x^2 - 22x - 14xy$

d. $2(t^3 + 1.4t^2 - 2.7t) - 4(0.5t^3 - t^2 + 1.3t)$ — $6.8t^2 - 10.6t$

5. Multiply and simplify:

a. $(x + y)(x + 4)$ — $x^2 + (4 + y)x + 4y$

b. $(3xy)(2x^2y + 3y^2x + 3xy)$ — $6x^3y^2 + 9x^2y^3 + 9x^2y^2$

c. $(x - y)(x^2 + y + 3)$ — $x^3 + xy + 3x - x^2y - y^2 - 3y$

d. $(p + 6q)(p^2 + 2pq + q^2)$ — $p^3 + 8p^2q + 13pq^2 + 6q^3$

6. Divide:
   a. $(24x^4y^2z^3) \div (-3x^3y^4z)$ -8xz^2/y^2

   b. $\left[x^2 + 2x^4 - 3x^3 + x - 2\right] \div \left[x^2 - 3x + 2\right]$ 2x^2 + 3x + 6 + (13x -14)/(x^2 -3x + 2)

7. Factor:
   a. $x^2 + xy$ x(x + y)

   b. $x^2 - y^2$ (x + y)(x – y)

   c. $4x^2 - y^2$ (2x + y)(2x – y)

   d. $x^2 - 7x + 6$ (x – 6)(x – 1)

   e. $x^2 + 2xy - 8y^2$ (x + 4y)(x – 2y)

   f. $6x^2y + 4y^2x + 2$ 2xy(3x + 2y) + 2

8. Simplify:

a. $\dfrac{x^2 - xy}{x^2 - 3x}$ (x – y)/(x – 3)

b. $\dfrac{x^2 - y^2}{(x+y)^2}$ (x – y)/x + y)

c. $\dfrac{x^2 - 3x + 2}{2 - x}$ (1 – x)

9. Express as a single fraction:

a. $\dfrac{1}{x} + \dfrac{4}{y}$ (4x + y)/xy

b. $\dfrac{4}{3xy} - \dfrac{5}{6yz}$ (8z – 5x)/6xyz

c. $\dfrac{6}{x^2 - 6} + \dfrac{3x}{x^2 + 2}$ $6(x^2 + 2) + 3x(x^2 - 6)/(x^2 - 6)(x^2 + 2)$

Name: ________________

Linear & Quadratic Equation Tutorial Solutions

1. Solve for x :

   a. $7x - 3 = 25$ — $x = 4$

   b. $2x + 1 = 3x - 3$ — $x = 4$

   c. $3(x + 7) - 2(x + 13) = 0$ — $x = 5$

   d. $\frac{x-2}{x+2} = \frac{x-4}{x+4}$ — $x = 0$

2. Solve for x and y

   a. $3x + 6y = 11$
   $14x - y = 3$ — $x = 1/3$; $y = 5/3$

   b. $-3y + 2x = 2$
   $3x + 5y = 41$ — $x = 7$; $y = 4$

   c. $3x - 1 = -y + 7$
   $x + 3y = 0$ — $x = 3$; $y = -1$

3. Solve for x, y, and z

   a. $$\begin{aligned} x + y + z &= 0 \\ 3x - 3y - 3z &= -12 \\ x - y + 2z &= -7 \end{aligned}$$ $x = -2$; $y = 3$; $z = -1$

b. $$\begin{aligned} 2x - y - 3z &= -11 \\ x - 2y - z &= -15 \\ 3x + 3y + z &= 26 \end{aligned}$$ x = 1; y = 7; z = 2

4. Solve for x by factorization:
   a. $x^2 + 3x + 2 = 0$ x = -1, -2

   b. $x^2 + 8x + 15 = 0$ x = -3, -5

   c. $x^2 - 6x + 9 = 11$ x = 3 ± √11

5. Solve for x using the standard quadratic formula:
   a. $3x^2 - 5x + 1 = 0$ x = (5 ± √13)/6

   b. $2x^2 - 6x + 3 = 0$ x = (6 ± √12)/4

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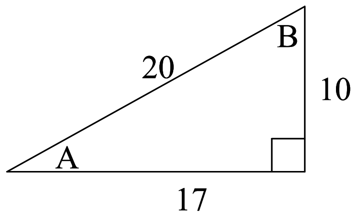
Trigonometry Tutorial

Solutions

1. Given:



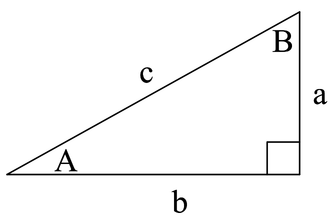
Find: sin A — 10/20=0.5

cos A — 17/20=0.85

tan A — 10/17=0.59

2. Given: sin A = 2/5

c = 5



Find: a — 2

b — $\sqrt{c^2 - a^2} = \sqrt{25-4} = \sqrt{21}$

$\angle B$ — $\sin^{-1}(b/c) = \sin^{-1}(0.92) = 66^\circ$

3. $\sin 45 = \frac{1}{\sqrt{2}}$ $\sin 60 = \frac{\sqrt{3}}{2}$

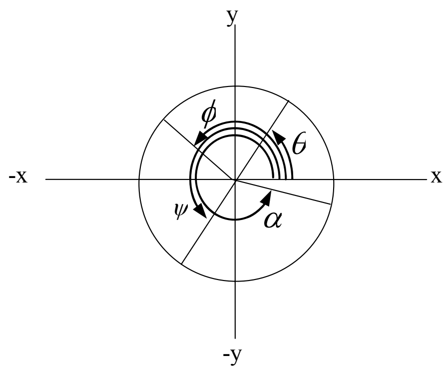
cos 45 = $\sqrt{2}/2$ cos 60 = 1/2

tan 45 = 1 sin 30 = 1/2

sin 0 = 0 tan 60 = $\sqrt{3}$

cos 0 = 1

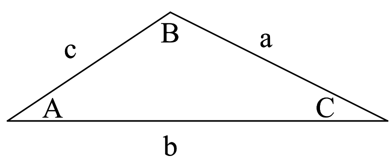
4. Given:



Find: sign of:

	$\tan\theta$	+
	$\sin\phi$	+
	$\cos\psi$	–
	$\sin\alpha$	–

5. Given:



$a = 126$

$\angle B = 60^o$

$\angle C = 65^o$

Find:

$\angle A$ — 180°-60°-65°=55°

b — $b = a\dfrac{sin(\angle B)}{sin(\angle A)} = 126\dfrac{0.87}{0.82} = 133.2$

c — $c = a\dfrac{sin(\angle C)}{sin(\angle A)} = 126\dfrac{0.91}{0.82} = 139.4$

[Recall the law of sines and/or the law of cosines]

6. Show that:

$$\cos(\alpha+2\beta)=\cos\beta(\cos\alpha\cos\beta-\sin\alpha\sin\beta)-\sin\beta(\cos\alpha\sin\beta+\sin\alpha\cos\beta)$$

$$\begin{aligned}\cos(\alpha+2\beta)&=\cos\alpha\cos(2\beta)-\sin\alpha\sin(2\beta)=\cos\alpha(\cos^2\beta-\sin^2\beta)-2\sin\alpha\sin\beta\cos\beta=\\&=\cos\alpha\cos^2\beta-\sin\alpha\sin\beta\cos\beta-\cos\alpha\sin^2\beta-\sin\alpha\sin\beta\cos\beta=\\&=\cos\beta(\cos\alpha\cos\beta-\sin\alpha\sin\beta)-\sin\beta(\cos\alpha\sin\beta+\sin\alpha\cos\beta)\end{aligned}$$

7. Find the corresponding number of radians or degrees

   a. 315 degrees $7\pi/4$

   b. 120 degrees $2\pi/3$

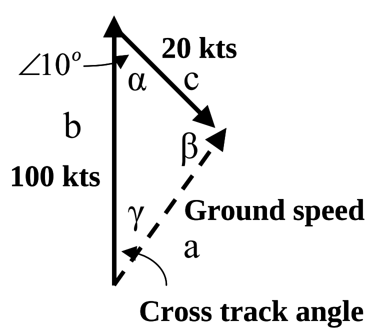
   c. 100 degrees $5\pi/9$

   d. π radians   180°

   e. $\frac{5\pi}{4}$ radians   225°

   f. 1.6 radians   91.72°

8. Given an aircraft traveling north at 100 kts into a 20 knot headwind from 350 degrees.

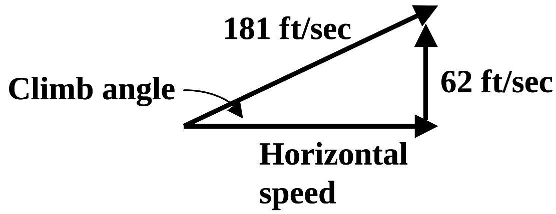


Find: Ground speed and cross track angle

$a^2 = b^2 + c^2 - 2bc\cos\alpha = 100^2 + 20^2 - 2(100)(20)(.985) = 6460$
GS = a = 80.4

$\sin\gamma = (c/a)\sin\alpha = (20/80.4)\sin(10°) = 0.043$
CTA = $\sin^{-1}(\gamma) = 2.477° = 2°\ 29'$

9. Given an aircraft flying at 181 ft/sec and climbing at 62 ft/sec.

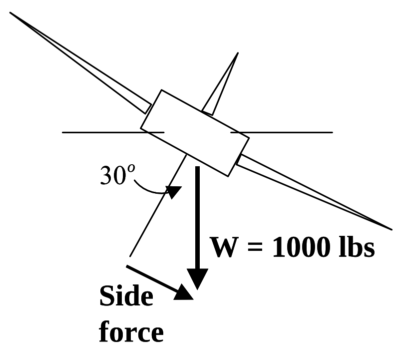


Find: Horizontal speed and climb angle.

$$HS = \sqrt{181^2 - 62^2} = 170\ ft/sec$$

$$CA = sin^{-1}\left(\frac{62}{181}\right) = 20^{\circ}$$

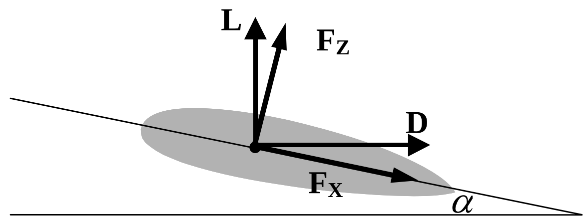
10. Given a 1000 lb aircraft in a 30 degree bank.



Find: Side force.

Side Force = 1000 sin (30°) = 500 lbs

11. Given:



Assuming small angle theory (α is small), why is $F_Z \approx L$ and $F_X \approx D$?

$\sin\alpha \approx 0$
$\cos\alpha \approx 1$

$F_X = -L\sin\alpha + D\cos\alpha \approx D$
$F_Z = L\cos\alpha + D\sin\alpha \approx L$

12. During roll performance testing, the F-99A rolled from 60 degrees left wing down to 60 degrees left wing up in 0.4 seconds.

Find the roll rate in:

Degrees/second $\Delta\Phi/\Delta t = 120°/0.4 = 300°/\text{sec}$

Radians/second 5.23 rads/sec

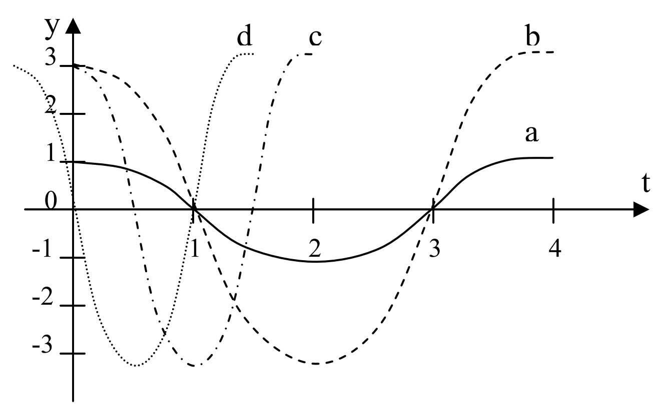
13. Plot in the same graph the following trigonometric functions:

a. $y = \cos\left(\frac{\pi}{2}t\right)$

b. $y = 3\cos\left(\frac{\pi}{2}t\right)$

c. $y = 3\cos(\pi t)$

d. $y = 3\cos\left(\pi t + \frac{\pi}{2}\right)$



What are the periods and frequencies (both in Hz and radians/sec) of those functions?

a. and b. T = 4 sec $\omega = 0.25$ Hz $= \pi/2$ rad/sec
c. and d. T = 2 sec $\omega = 0.5$ Hz $= \pi$ rad/sec

Pre-TPS

Co-ordinate Systems and Graphs Solutions

1. a. +8; b. – 1; c. +3

2. L_1 is y = 2x; L_2 is y = x + 1; P_1 is (1, 2)

3.

    a. Slope $P_2 - P_1 = -1$. Slope $P_1 - P_3 = -1$. Equation of line through P_1P_2 is y = -x + 1. This is satisfied by P3, (2, -1). Therefore all lie on same straight line.

    b. Slope $P_1 - P_2 = 2$. Slope $P_2 - P_3 = 2$. Slope $P_3 - P_4 = 2$. Equation of line through P_2 with slope = 2 is y = 2x + 3. P_1, P_3 and P_4 all satisfy this. Therefore all are on same straight line.

4. Slope $P_1 - P_2 = ¼$. Slope $P_2 - P_3 = -4$. $m_1.m_2 = -1$. Therefore P_1P_2, P_2P_3 are perpendicular. Therefore $P_1P_2P_3$ is a right-angled triangle.

5. a. 3. b. -1. c. - ¾.

6. 2y = x + 3

7. $y = \sqrt{3}.x + (4 - \sqrt{3})$

8. a. Slope of both is A/B therefore both are parallel. b. Slope of one is A/B, slope of other is -B/A. Product of slopes is -1, therefore they are perpendicular.

9. C = 5/9(F – 32). The scales are numerically equal at -40^o.

10.

    a. Center at (0, 1), radius = 2

    b. Center at (-1, 0), radius = 3

    c. Center at (-1, 2), radius = 2

11.

    a. $x^2 = 8y$

    b. $x^2 + 4x = 4y - 16$; c. $x^2 - 2x = 12y + 35$.

Logarithms, Radicals and Exponents
Tutorial

1. Evaluate the following:

a. $\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^2 =$ $(1/2)^{3+2} = (1/2)^5$

b. $\frac{a^{10}}{a^4} =$ $a^{10-4} = a^6$

c. $\left(a^{n+2}\right)\left(a^{m+3}\right) =$ $a^{(m+n+5)}$

d. $\left(a^2\right)^5 =$ $a^{2x5} = a^{10}$

e. $\left(a^{2n}\right)^3 =$ a^{6n}

f. $\left(a^3 + b^5\right)^0 =$ 1

2. Evaluate the following:

a. $4^{2/3}$ $\sqrt[3]{4^2} = \sqrt[3]{16}$

b. $8^{-2/3}$ $\frac{1}{\sqrt[3]{8^2}} = \frac{1}{\sqrt[3]{64}}$

c. $\left(x^5\right)^{-4}$ $x^{-20} = 1/x^{20}$

d. $\left(a^{2/3}\right)^{3/4}$ $\sqrt{a}$

e. Expand $\sqrt[7]{x^2 y^5}$ $\sqrt[7]{x^2}\sqrt[7]{y^5}$

f. Simplify $\sqrt{27}$ $\sqrt{9}\sqrt{3} = 3\sqrt{3}$

3. Write the following in logarithmic form:

a. $7^2 = 49$ — $\text{Log}_7 49 = 2$

b. $3^2 = 27$ — $\text{Log}_3 27 = 3$

c. $2^{-3} = \frac{1}{8}$ — $\text{Log}_2(1/8) = -3$

d. $\sqrt[3]{8} = 2$ — $\text{Log}_8 2 = 1/3$

4. Write the following in exponential form:

a. $\log_3 81 = 4$ — $3^4 = 81$

b. $\log_9 27 = \frac{3}{2}$ — $9^{3/2} = 27$

c. $\log_{10} 50 = 1.699$ — $10^{1.699} = 50$

5. Simplify the following:

$\log_{10}(5)(9) + \log_{10}\frac{25}{9} - \log_{10} 5$

Log5 + Log9 + Log25 – Log9 – Log5

= Log25

6. Find:

   a. $\log_{10} 3860$ 3.5866

   b. $\log_{10} 5.46$ 0.7372

   c. $\log_{10} .00235$ -2.6289

   d. $\log_{10} .0000129$ -4.8894

   e. $\log_{10} 72800$ 4.8621

7. Solve for x:

   a. $(3)(10^x) = 27$ 0.9542

   b. $e^{2(x-5)} = 30$ 6.7

   c. $2e^x = 8$ 1.3863

   d. ln x – ln (x-1) = 2 1.156

Complex Numbers Tutorial
Solutions

1. Perform the indicated operations:

a. (3 – 4i) – (-5 + 7i)	8 – 11i
b. (4 + 2i) + (-1 + 3i)	3 + 5i
c. (2 + i)(3 + 2i)	4 + 7i
d. (3 – 4i)(3 + 4i)	25
e. $\frac{1+3i}{2-i}$	–1/5 + (7/5)i
f. $\frac{3-2i}{2+3i}$	–i

2. Find the conjugate of the following:

a. 2 + i	2 – i
b. 2 – 3i	2 + 3i
c. –4 + 2i	–4 – 2i
d. –4 – 3i	–4 + 3i
e. 3i – 7	–7 – 3i

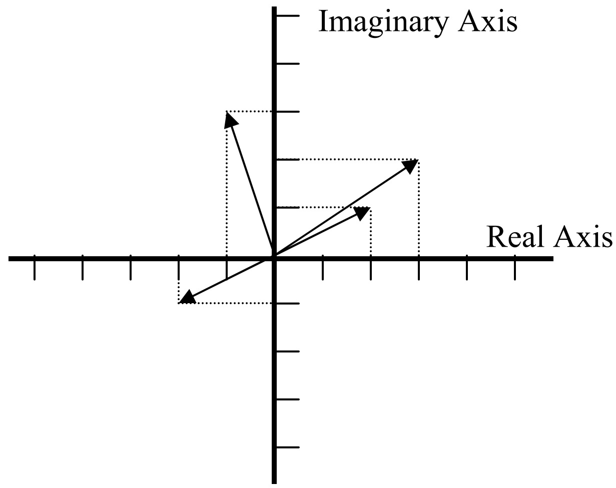
3. Graph the following:

    a. 3 + 2i

    b. 2 + i

    c. –2 – i

    d. –1 + 3i

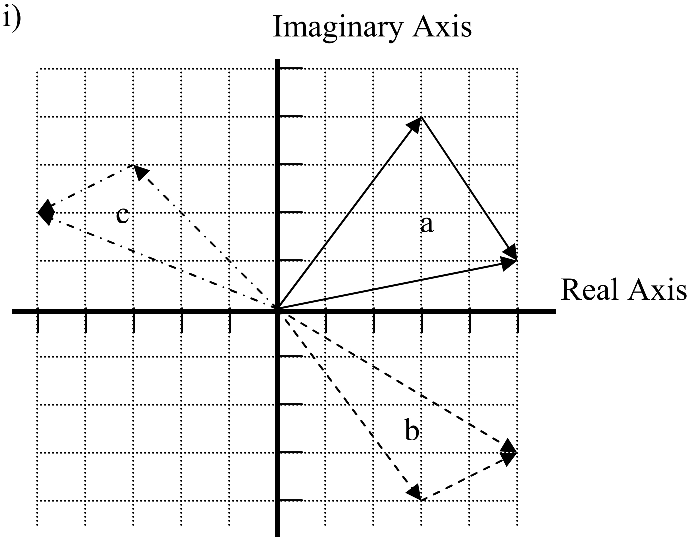


4. Graphically add the following:

    a. (3 + 4i) + (2 – 3i)

    b. (3 – 4i) + (2 + i)

    c. (–3 + 3i) – (2 + i)



5. Express the following in polar form:

a. $+1+i\sqrt{3}$

2 (cos60°+ i sin60°)

$r=\sqrt{1+3}=2$

$\theta=tan^{-1}\left(\sqrt{3}/1\right)=60^{\circ}$

b. $6\sqrt{3}+6i$

12 (cos30°+ i sin30°)

$r=\sqrt{36\cdot 3+36}=12$

$\theta=tan^{-1}\left(6/6\sqrt{3}\right)=30^{\circ}$

c. 0 + 4i

4 (cos90°+ i sin90°)

$r=\sqrt{0+16}=4$

$\theta=tan^{-1}(4/0)=90^{\circ}$

d. -1 + i

√2 (cos135°+ i sin135°)

$r=\sqrt{1+1}=\sqrt{2}$

$\theta=tan^{-1}(1/-1)+180^{\circ}=-45^{\circ}+180^{\circ}=135^{\circ}$

6. Express the following in rectangular form:

a. 4(cos 60° + i sin 60°)

$2+2\sqrt{3}i$

b. 3(cos 90° + i sin 90°)

$0+3i$

c. 2(cos 45° + i sin 45°)

$\sqrt{2}+\sqrt{2}i$

7. Use De Moivre's theorem to evaluate the following and express results in a + bi form:

a. $\left(1+\sqrt{3}i\right)^5$ $\underline{16-16\sqrt{3}i}$

$$\left(1+\sqrt{3}i\right)^5=\left[2\left(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}\right)\right]^5=32\left(\cos\frac{5\pi}{3}+i\sin\frac{5\pi}{3}\right)=$$
$$=32\left(\frac{1}{2}-i\frac{\sqrt{3}}{2}\right)=16-16\sqrt{3}i$$

b. $\sqrt{\left(1-\sqrt{3}i\right)}$ $\underline{\sqrt{6}-\sqrt{2}i}$

$$\sqrt{\left(1-\sqrt{3}i\right)}=\left(1-\sqrt{3}i\right)^{1/2}=\left\{2\left[\cos\left(-\frac{\pi}{3}\right)+i\sin\left(-\frac{\pi}{3}\right)\right]\right\}^{1/2}=$$
$$=\sqrt{2}\left[\cos\left(-\frac{\pi}{6}\right)+i\sin\left(-\frac{\pi}{6}\right)\right]=$$
$$=\sqrt{2}\left(\frac{\sqrt{3}}{2}-i\frac{1}{2}\right)=\frac{\sqrt{6}}{2}-\frac{\sqrt{2}}{2}i$$

8. Express the following in the alternate forms requested:

a. 4(cos 60° + i sin 60°) exponential form: $\underline{4e^{i\pi/3}}$

b. $6\sqrt{3}+6i$ exponential form: $\underline{12e^{i\pi/6}}$

$$r=\sqrt{36\cdot 3+36}=12$$
$$\theta=\tan^{-1}\left(\frac{6}{6\sqrt{3}}\right)=\frac{\pi}{6}$$

e. $4e^{\frac{\pi}{2}i}$ polar form: $\underline{4\ (\cos 90°+i\sin 90°)}$

$$4e^{\frac{\pi}{2}i}=4\left(\cos\frac{\pi}{2}+i\sin\frac{\pi}{2}\right)=$$
$$=4(0+i)$$

rectangular form: $\underline{0+4i}$

Determinant & Matrix
Tutorial

1. Solve the following determinants:

a. $\begin{vmatrix} 2 & 4 \\ 3 & 5 \end{vmatrix}$ -2

b. $\begin{vmatrix} -3 & -4 \\ 2 & 7 \end{vmatrix}$ -13

c. $\begin{vmatrix} 1 & 1 & 1 \\ 3 & -3 & -3 \\ 1 & -1 & 2 \end{vmatrix}$ -18

d. $\begin{vmatrix} 3 & -1 & 1 \\ 5 & 6 & 4 \\ 0 & 1 & 2 \end{vmatrix}$ 39

2. Solve the following using Cramer's Rule

$x + y + z = 0$ $\qquad$ $x = 2$

$3x – 3y - 3z = 12$ $\qquad$ $y = 5/3$

$x –y + 2z = -7$ $\qquad$ $z = -11/3$

3. Add or subtract the following matrices

a. $\begin{bmatrix} 2 & 4 \\ 3 & 5 \end{bmatrix} + \begin{bmatrix} -3 & -4 \\ 2 & 7 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 5 & 12 \end{bmatrix}$

b. $\begin{bmatrix} 1 & 1 & 1 \\ 3 & -3 & -3 \\ 1 & -1 & 2 \end{bmatrix} - \begin{bmatrix} 3 & -1 & 1 \\ 5 & 6 & 4 \\ 0 & 1 & 2 \end{bmatrix} = \begin{bmatrix} -2 & 2 & 0 \\ -2 & -9 & -7 \\ 1 & -2 & 0 \end{bmatrix}$

4. Multiply the following

$$3\begin{bmatrix} 3 & -1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 5 & 6 & 4 \\ 1 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 42 & 51 & 30 \\ 3 & 3 & 6 \end{bmatrix}$$

5. Transpose

$$\begin{bmatrix} 3 & 1 \\ 2 & 4 \\ 7 & 5 \end{bmatrix} = \begin{bmatrix} 3 & 2 & 7 \\ 1 & 4 & 5 \end{bmatrix}^T$$

6. Write the following set of equations in matrix form

2x + 7y = 26

5x – 2y = 14

$$\begin{bmatrix} 2 & 7 \\ 5 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 26 \\ 14 \end{bmatrix}$$

7. Solve for x, y, and z [Hint: Cramer]

$$\begin{bmatrix} 1 & 2 & 1 \\ 3 & -4 & -2 \\ 5 & 3 & 5 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 2 \\ 1 \end{bmatrix}$$

$$x = \frac{-70}{-35}$$

$$y = \frac{-95}{-35}$$

$$z = \frac{120}{-35}$$

Vector Algebra Tutorial
Solutions

1. Length of vector $= \sqrt{\left(\frac{1}{\sqrt{3}}\right)^2 + \left(\frac{1}{\sqrt{3}}\right)^2 + \left(\frac{1}{\sqrt{3}}\right)^2}$

   $= \sqrt{\frac{1}{3} + \frac{1}{3} + \frac{1}{3}} = 1$

2. $\sqrt{a^2 + b^2 + c^2} = \sqrt{4 + 9 + 1} = \sqrt{14}$

   unit vector is $\frac{2}{\sqrt{14}}\hat{i} + \frac{3}{\sqrt{14}}\hat{j} - \frac{1}{\sqrt{14}}\hat{k}$

3. Length of $\overline{A} = \sqrt{14}$
   Length of $\overline{B} = \sqrt{56}$
   } UNEQUAL

4. $\overline{F}_R = 2\hat{i} - \hat{j}$ $\quad \left|\overline{F}_R\right| = \sqrt{5}$

5. a. $2\overline{A} - \overline{B} + 3\overline{C} = 11\hat{i} - 8\hat{k}$

   b. $\left|\overline{A} + \overline{B} + \overline{C}\right| = \sqrt{93}$

6. a. $\overline{A} \cdot \overline{B} = 2$

   b. $\left|\overline{A}\right| = \sqrt{26}$ $\quad \left|\overline{B}\right| = \sqrt{29}$

7. $\overline{A} \cdot \overline{B} = 28$

   $\left|\overline{A}\right| = \sqrt{14}$ $\quad \left|\overline{B}\right| = \sqrt{56}$

   $\therefore 28 = \sqrt{14} \cdot \sqrt{56} \cos\theta$

   $\cos\theta = \frac{28}{\sqrt{14} \cdot \sqrt{56}} = 1 \qquad \theta = 0^\circ$

8. –3

9. $a_x = 3$, $a_y = -1$, $a_z = -4$; $\quad b_x = -2$, $b_y = 4$, $b_z = -3$

   $$\overline{A} \times \overline{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -1 & -4 \\ -2 & 4 & -3 \end{vmatrix} = \begin{vmatrix} -1 & -4 \\ 4 & -3 \end{vmatrix}\hat{i} + (-1)\begin{vmatrix} 3 & -4 \\ -2 & -3 \end{vmatrix}\hat{j} + \begin{vmatrix} 3 & -1 \\ -2 & 4 \end{vmatrix}\hat{k}$$

   $= 19\hat{i} + 17\hat{j} + 10\hat{k}$

10. Proceed as in 9
11. And again
12. a. $-8\hat{i} - 6\hat{k}$

    b. $2\hat{i} - \hat{j}$

Pre-TPS
Differentiation
Solutions

1. *a.* $f(x) = 2x^2 + x$ — 4x + 1

   b. $f(x) = 3x^2 + 2x + 1$ — 6x + 2

2. a. $\frac{d}{dx}\left(2x^2 + x\right)$ — 4x + 1

   b. $\frac{d}{dx}\left(3x^3 - 4x^2 + 5x - 2\right)$ — $9x^2$ - 8x + 5

   c. $\frac{d}{dx}\left(2u^2 v\right)$ where *u* and *v* are functions of *x* — $2u^2$(dv/dx) + 4uv(du/dx)

   d. $\frac{d}{dx}\left(1/2x^4 + 5x\right)$ — $2x^3 + 5$

   e. $\frac{d}{dx}\left(2u^2 / v^3\right)$ where *u* and *v* are functions of *x* — $\dfrac{4uv\dfrac{du}{dx} - 6u^2\dfrac{dv}{dx}}{v^4}$

3. $y = 2z^2 + z$ and $z = (x - 2)$; chain rule gives $\frac{dy}{dx}$ = (4z + 1).1

4. $y = x^3 + 4x + 3$; $\frac{dy}{dx} = 3x^2 + 4$

5. $y = \frac{x^2 - 3}{x + 4}$; $\frac{dy}{dx} = \frac{x^2 + 8x + 3}{(x + 4)^2}$

6. $y^2 + x - 4 = 0$; $\frac{dy}{dx}$ = -1/2y

7. $x^2 + 2xy - 3y^2 + 11 = 0$; $\frac{dy}{dx} = \frac{(x + y)}{(3y - x)}$; and so at the point (2,3) = 5/7

8. $\frac{d^2y}{dx^2}$ for the following are:

   a. $y = 3x^4 - 2x^3 + 6$ — $12(3x^2 - x)$

   b. $y = 4ax^{1/2}$ — $-ax^{-3/2}$

c. $y = (x + 2)(x - 3)$ 2

d. $y - x^2 - 12 = x^7 + 3x^4 + 4x^2 - x + 10$ $42x^5 + 36x^2 + 10$

9. $s = 120t - 16t^2$

velocity, $ds/dt = 120 - 32t$; acceleration, $d^2s/dt^2 = -32$

velocity at t = 2 = 56 ; acceleration at t = 2 = -32

10. maximum and minimum values for x and y below are:

a. $y = x^3 + 2x^2 - 15x - 20$ x = -3 or +5/3; y = +16 or -34.81

b. $y = x^2 - 10$ x = 0; y = -10

PreTPS
Integration
Solutions

1. Integrate the following non-definite integrals

a.	$\int \left(x^3 + 6x^2 + 7\right)dx$	$x^4/4 + 2x^3 + 7x + C$
b.	$\int \frac{dx}{x^2}$	$-1/x + C$
c.	$\int \frac{2x+1}{\left(x^2 + x\right)}dx$	$\mathrm{Ln}(x^2 + x) + C$
d.	$\int \sin 3x\, dx$	$\frac{-1}{3}\cos 3x + C$
e.	$\int \left(\cos 4x + \sec^2 x\right)dx$	$\frac{1}{4}\sin 4x + \tan x + C$
f.	$\int e^{3x}\, dx$	$\frac{1}{3}e^{3x} + C$

2. Evaluate the following definite integrals:

a.	$\int_0^{\frac{\pi}{2}} 3\sin x\, dx$	+3
b.	$\int_{-\pi}^{+\pi} 2\cos x\, dx$	0
c.	$\int_0^3 \left(x^2 + 7x + 6\right)dx$	117/2

3. Integrate by parts:

$\int x \cdot \sin x\, dx$ — Sinx – xCosx + C

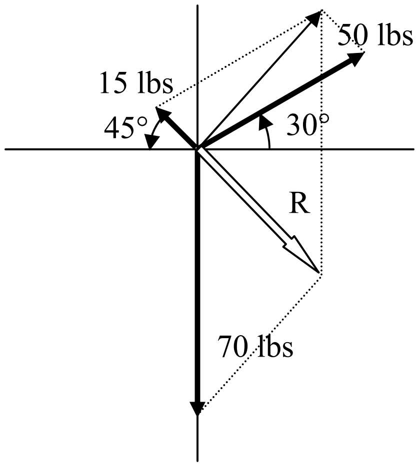
4. Integrate by substitution:

$\int \sin^3 x \cos x\, dx$ — ¼ $\mathrm{Sin}^4 x + C$

5. Find the area under the curve $y = x^3 + 3x^2 + 2$ between $x = 0$ and $x = 2$. 16 sq units

Statics and Friction Tutorial
Solutions

1. Given:



Find the resultant force (magnitude and angle)

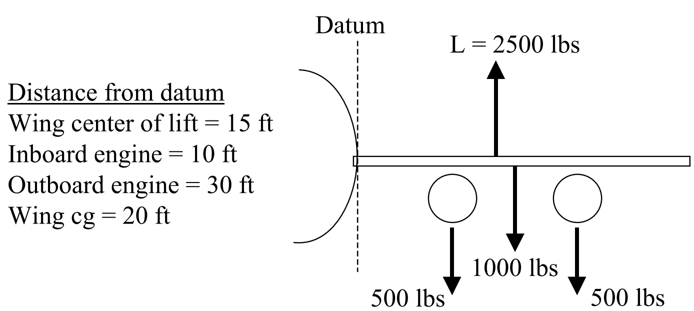
$$R_X = 50\cos 30^\circ - 15\cos 45^\circ = 50\frac{\sqrt{3}}{2} - 15\frac{\sqrt{2}}{2} = 32.7lbs$$

$$R_Y = 50\sin 30^\circ + 15\sin 45^\circ - 70 = 50\frac{1}{2} + 15\frac{\sqrt{2}}{2} - 70 = -34.4lbs$$

$$R = \sqrt{R_X^2 + R_Y^2} = 47.5lbs$$

$$\theta = tan^{-1}\left(\frac{R_Y}{R_X}\right) = -46.5^\circ$$

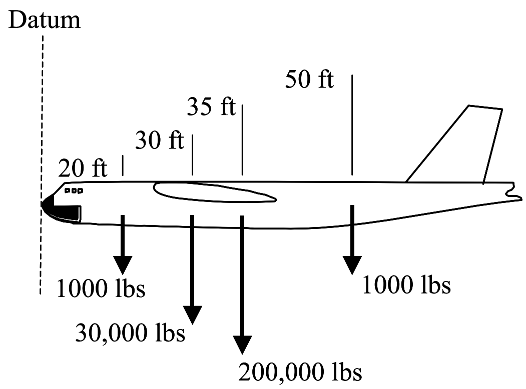
2. Given:



Find: $\sum M$ around datum

$$\sum M = 10 \cdot 500 + 20 \cdot 1{,}000 + 30 \cdot 500 - 15 \cdot 2{,}500 =$$
$$= 5{,}000 + 20{,}000 + 15{,}000 - 37{,}500 =$$
$$= 2{,}500\ ft \cdot lbs$$

3. Given:



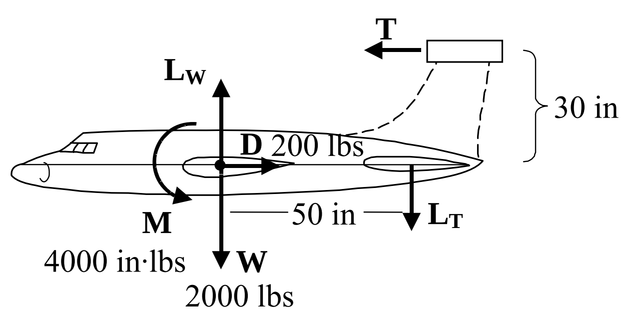
Find: a) Resultant force

$$\sum F = 1{,}000 + 30{,}000 + 200{,}000 + 1{,}000 = 232{,}000 lbs$$

b) Distance from datum to resultant force

$$d = \frac{\sum M}{\sum F} = \frac{20 \cdot 1{,}000 + 30 \cdot 30{,}000 + 35 \cdot 200{,}000 + 50 \cdot 1{,}000}{232{,}000} = 34{,}35\ ft$$

4. Given:



Find the following to keep the aircraft balanced.

a. T $\quad T = D = 200 lbs$

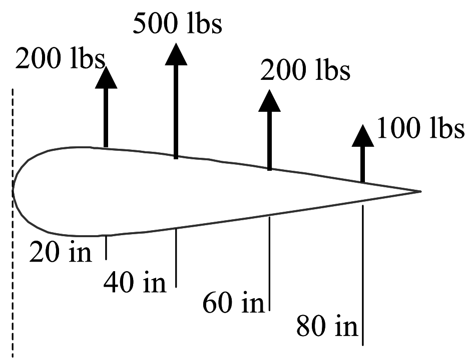
b. L_T

$$50 \cdot L_T = 30 \cdot T + 4{,}000$$

$$L_T = \frac{4000 + 30 \cdot 200}{50} = 200 lbs$$

c. L_W $\quad L_W = W + L_T = 2{,}200 lbs$

5. Given:



a. Find the resultant force (F_R)

$$F_R = \sum F = 1{,}000lbs$$

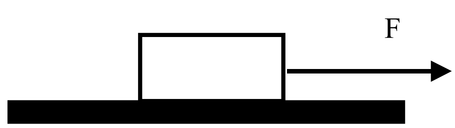
b. Find the distance from the leading edge to the resultant force ($\bar{x}$).

$$d = \frac{\sum M}{\sum F} = \frac{20 \cdot 200 + 40 \cdot 500 + 60 \cdot 200 + 80 \cdot 100}{1{,}000} = 44in$$

c. Transfer the resultant force to the 25 inch point and determine the resultant moment .

$$M_R = F_R \cdot d = 1{,}000 \cdot (44 - 25) = 19{,}000lb \cdot in$$

6. Given:

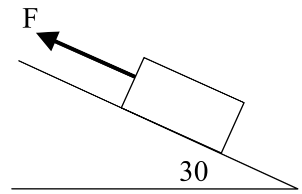


Weight of block = 112 lbs
μ_s= 0.25

Find the minimum force required to move the block.

$$F = \mu_s N = 0.25 \cdot 112 = 28lbs$$

7. Given:



Weight of block = 150 lbs

μ_s= 0.3

Find: a) Minimum force required to hold the block at rest.

$$N = W\cos 30^\circ = 150\frac{\sqrt{3}}{2} = 130lbs$$

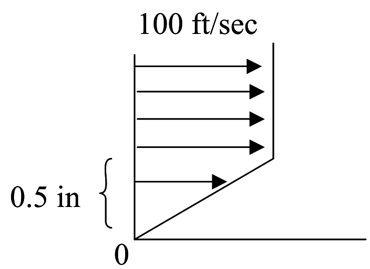
$$f_{max} = \mu_s N = 39lbs$$

$$F_{min} = W\sin 30^\circ - f_{max} = 75 - 39 = 36lbs$$

b) Maximum force required to hold the block at rest.

$$F_{max} = W\sin 30^\circ + f_{max} = 75 + 39 = 114lbs$$

8. Given:



Calculate $\frac{dV}{dy}$ from this data

$$\mu = 1.2x10^{-5}\frac{lb - \sec}{ft^2}$$

Find: a) $\frac{dV}{dy}$

$$\frac{dV}{dy} = \frac{100\ ft/sec}{0.5in} = \frac{100\ ft/sec}{0.5\ ft/12} = 2{,}400/sec$$

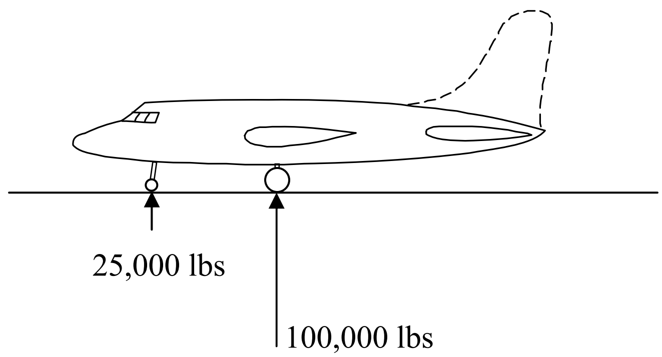
b) τ

$$\tau = \mu\frac{dV}{dy} = 28.8\cdot 10^{-3}\frac{lb}{ft^2}$$

c) Shear force acting over a 200 ft^2 area

$$F = \tau\cdot S = 28.8\cdot 10^{-3}\cdot 200 = 5.76lb$$

9. Consider an aircraft weighing 125,000 lbs taxing on the ground.



Assuming that:

- the reaction force on the nose wheel is 25,000 lbs;
- the reaction force on the main gear is 100,000 lbs (50,000 lbs per wheel)
- the radius of the nose wheel is 25 in;
- the radius of the main gear wheel is 50 in;
- the coefficient of rolling resistance is b=1 in;
- the aerodynamic drag is negligible;

find the engine thrust necessary to maintain a constant ground speed

$$F_{nose} = N_{nose} \cdot \frac{b}{r_{nose}} = 25{,}000\frac{1}{25} = 1{,}000 lbs$$

$$F_{main} = 2\left(\frac{N_{main}}{2}\frac{b}{r_{main}}\right) = 100{,}000\frac{1}{50} = 2{,}000 lbs$$

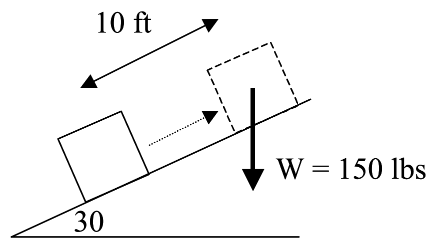
$$T = F_{nose} + F_{main} = 3{,}000 lbs$$

10. Assuming μ_s=0.4, calculate the maximum braking force the crew can apply without skidding.

$$f = \mu_s N_{main} = 0.4 \cdot 100{,}000 = 40{,}000 lbs$$

Work and Energy Tutorial
Solutions

1. Determine the amount of work performed on the block when it is moved 10 ft UP the incline as shown. ASSUME NO FRICTION



$$W \sin\left(30^\circ\right) \cdot 10 = 150 \cdot \frac{1}{2} \cdot 10 = 750\ ft \cdot lb$$

2. Determine the amount of work done in #1 above if μ =0.2

$$\left[W \sin\left(30^\circ\right) + \mu W \cos\left(30^\circ\right)\right] \cdot 10 = \left(150 \cdot \frac{1}{2} + 0.2 \cdot 150 \cdot \frac{\sqrt{3}}{2}\right) \cdot 10 = 1010\ ft \cdot lb$$

3. What is the potential energy of a 240,000 lb aircraft flying at 36,000 ft and 380 kts? [(kts)(1.68) = ft/sec]

$$PE = W \cdot H = 240{,}000 \cdot 36{,}000 = 864 \cdot 10^7\ ft \cdot lb$$

What is the kinetic energy?

$$KE = \frac{1}{2}\frac{W}{g}V^2 = \frac{1}{2}\frac{240{,}000}{32.2}\left(380 \cdot 1.68\right)^2 = 152 \cdot 10^7\ ft \cdot lb$$

What is the total specific energy?

$$SE = \frac{TE}{W} = \frac{\left(864 + 153\right) \cdot 10^7}{240{,}000} = 42{,}330\ ft$$

4. A 10,000 lb fighter experiences an engine flame-out at 30,000 ft and 1200 ft/sec airspeed. Assuming no energy losses during a zoom climb, calculate the maximum altitude when the aircraft reaches 500 ft/sec velocity.

$$SE = H + \frac{1}{2}\frac{V^2}{g} = 30{,}000 + \frac{1}{2}\frac{1{,}200^2}{32.2} = h_{max} + \frac{1}{2}\frac{500^2}{32.2}$$

$$h_{max} = 30{,}000 + \frac{1}{64.4}\left(1{,}200^2 - 500^2\right) = 48{,}478\ ft$$

5. An aircraft is flying at 35,000 ft and 1000 ft/sec airspeed. The aircraft weighs 35,000 lbs.
   a. Find the specific energy of the aircraft

$$SE = H + \frac{1}{2}\frac{V^2}{g} = 35{,}000 + \frac{1}{2}\frac{1{,}000^2}{32.2} = 50{,}528\ ft$$

   b. Assuming no losses, find the maximum velocity of the aircraft at sea level.

$$SE = \frac{1}{2}\frac{V^2}{g} = 50{,}528\ ft$$

$$V = \sqrt{2 \cdot 32.2 \cdot 50{,}528} = 1804\ ft/sec$$

6. A spring is compressed 6 inches. (K = 300 lb/in) If a 50 lb object is placed on top of the compressed spring and the spring is released.
   a. What is the spring force before release?

$$F = K \cdot x = 300 \cdot 6 = 1800 lb$$

   b. Calculate the stored energy in the spring before it is released.

$$E = \frac{1}{2}Kx^2 = \frac{1}{2} \cdot 300 \cdot 36 = 5400 in \cdot lb = 450\ ft \cdot lb$$

   c. What is the velocity of the object at separation from the spring. (assume Stored energy = KE)

$$\frac{1}{2}\frac{W}{g}V^2 = \frac{1}{2}Kx^2 = 450\ ft \cdot lb$$

$$V = \sqrt{\frac{2 \cdot g \cdot 450}{W}} = \sqrt{\frac{2 \cdot 32.2 \cdot 450}{50}} = 24\ ft/sec$$

Kinematics
Tutorial Solutions

Take g as 32 ft/sec^2

1. A train's speed increases uniformly from 30 mi/hr to 60 mi/hr in 5 minutes. Determine the average speed, the distance traveled and the acceleration. (66 ft/sec; 19,800 ft; 0.147ft/sec^2)

2. A stone dropped from a tower strikes the ground in 3 sec. Determine the height of the tower. (144 ft)

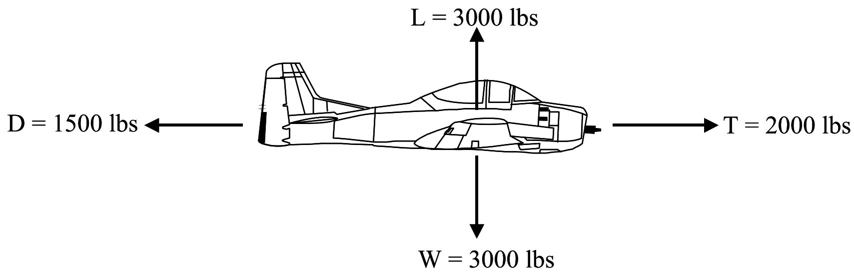
3. A stone is thrown vertically upwards with a velocity of 96 ft/sec. Calculate the time taken to reach the highest point; the greatest height reached; and the total time before the stone hits the ground. (3 sec; 144 ft; 6 sec)

4. A 3lb body is whirled on a 4ft string in a horizontal circle. Calculate the tension in the string if the speed is (a) 8 ft/sec, (b) 2 RPS. (1.5 lb; $6\pi^2$ lb)

5. A body rests in a pail which is moved in a vertical circle of radius 2ft. What is the least speed the body must have so as not to fall out when at the top of the path? (8ft/sec)

Newton's Laws
Tutorial Solutions

1.



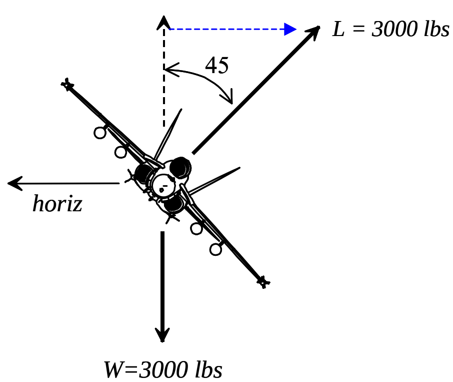
a. Is the aircraft in steady, unaccelerated flight? Why?

No. Net horizontal forward force 500 lbs

b. Calculate the aircraft acceleration

$5.33 ft/sec^2$

2. T = D = 2000 lbs



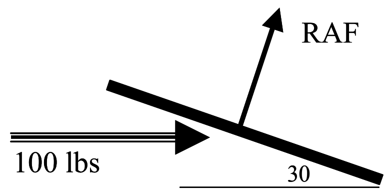
a. Is the aircraft in level flight? Why?

No. Vertical forces not balanced.

b. Calculate the vertical acceleration

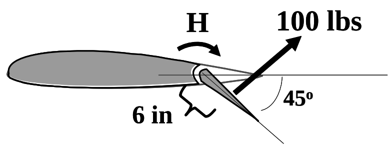
$9.6 ft/sec^2$ descent

3. Compute RAF



RAF = 50 lbs

4.



a. In order to hold the elevator in place how much hinge moment (H) is required?

+25 ft-lbs

b. If the elevator hinge suddenly breaks, what will be the horizontal acceleration of the elevator surface [Assume the elevator weights 10 lbs]

227.6 ft/sec^2

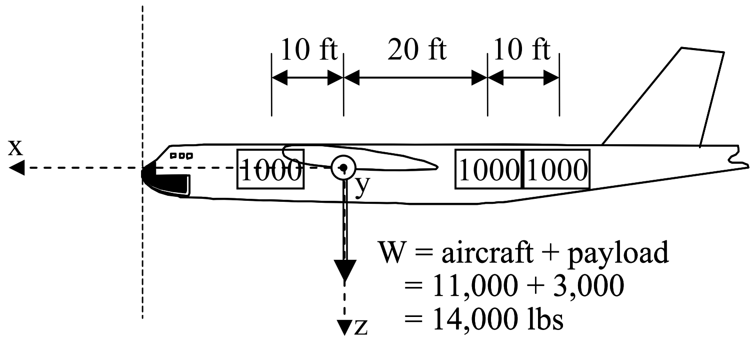
5. An aircraft weighing 20,000 lbs (including payload) drops a 5000 lb bomb from straight, level unaccelerated flight. Calculate the vertical acceleration of the aircraft immediately after dropping the bomb.

10.7 ft/sec^2

Inertia Tutorial

Solutions

1.



a. Find I_y for the aircraft loaded as shown above (empty weight moment of inertia: $(I_y)_{empty}$=80,000 slug·ft^2).

$$(I_Y)_{load} = \frac{1{,}000}{32.2}\left(10^2 + 20^2 + 30^2\right) = 43{,}478 slug \cdot ft$$

$$I_Y = (I_Y)_{empty} + (I_Y)_{load} = 123{,}478 slug \cdot ft$$

b. Find I_y of the aircraft after the 1000 lb payload is dropped from the forward bay.

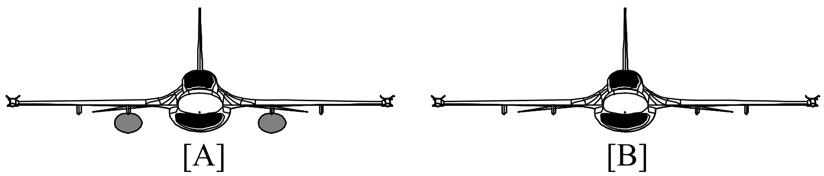
$$I_Y = 123{,}478 - \frac{1{,}000}{32.2}10^2 = 120{,}372 slug \cdot ft$$

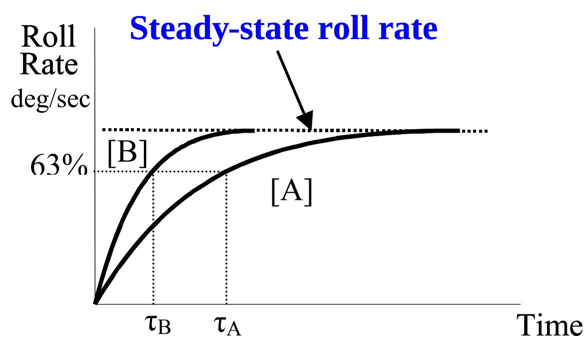
c. If the short period frequency of the aircraft is $f\left(\frac{1}{I_y}\right)$, does the short period frequency of the aircraft increase, decrease, or stay the same after the forward payload is released?

Increase

2. The roll mode (τ) time constant is a measure of how quickly the maximum roll rate (p) can be reached.

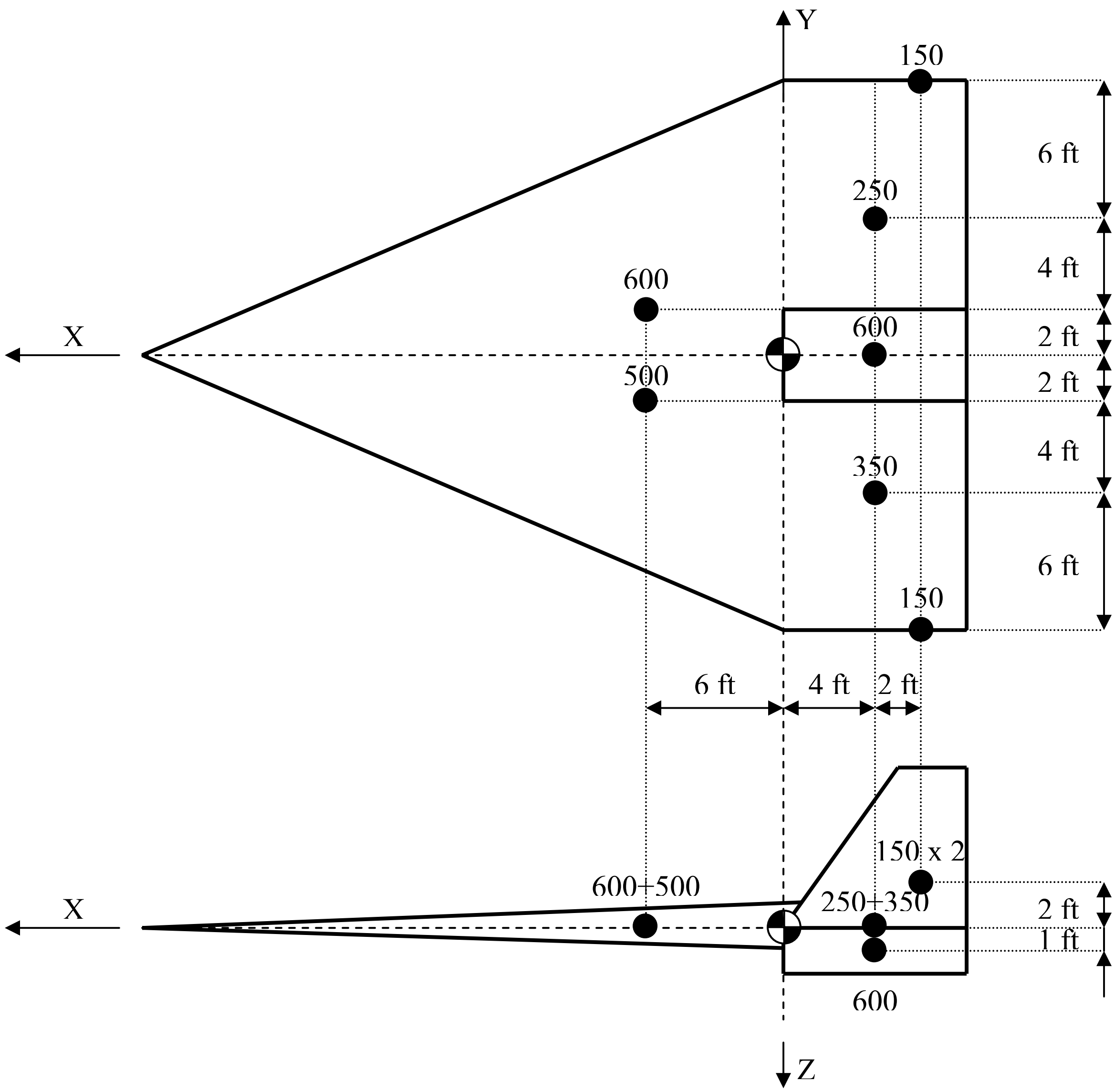
$$\tau = f\left(\frac{I_x}{damping}\right)$$





Match the roll mode time constant with the appropriate configuration.

3. Given the following future X-airplane, calculate the moments of inertia (Ix, Iy, Iz) and the products of inertia (Ixy, Iyz, Ixz).



$$I_X = (600+500)\cdot 2^2 + (250+350)\cdot 6^2 + (150\cdot 2)\cdot (12^2+2^2) + 600\cdot 1^2 = 71{,}000 slug \cdot ft^2$$

$$I_Y = (600+500)\cdot 6^2 + (250+350)\cdot 4^2 + (150\cdot 2)\cdot (6^2+2^2) + 600\cdot (4^2+1^2) = 71{,}400 slug \cdot ft^2$$

$$I_Z = (600+500)\cdot (6^2+2^2) + (250+350)\cdot (4^2+6^2) + (150\cdot 2)\cdot (6^2+12^2) + 600\cdot 4^2 = 138{,}800 slug \cdot ft^2$$

$$I_{XY} = 600\cdot 6\cdot 2 + 500\cdot 6\cdot (-2) + 250\cdot (-4)\cdot 6 + 350\cdot (-4)\cdot (-6) + 150\cdot (-6)\cdot 12 + 150\cdot (-$$

$$= 3{,}600 slug \cdot ft^2$$

$$I_{YZ} = 150\cdot 12\cdot (-2) + 150\cdot (-12)\cdot (-2) = 0 slug \cdot ft^2$$

$$I_{XZ} = (150\cdot 2)\cdot (-6)\cdot (-2) + 600\cdot (-4)\cdot 1 = 1{,}200 slug \cdot ft^2$$

Momentum and Impulse
Tutorial Solutions

1. An 8gm bullet is fired horizontally into a 9kg block of wood which is free to move. The velocity of the block and bullet after impact is 40cm/sec. Calculate the initial velocity of the bullet. (V = 45,040cm/sec)

2. A 600lb gun mounted on wheels fires a 10lb projectile with a muzzle velocity of 1800ft/sec at an angle of 30^o above the horizontal. Calculate the horizontal recoil velocity of the gun. (V = 26ft/sec)

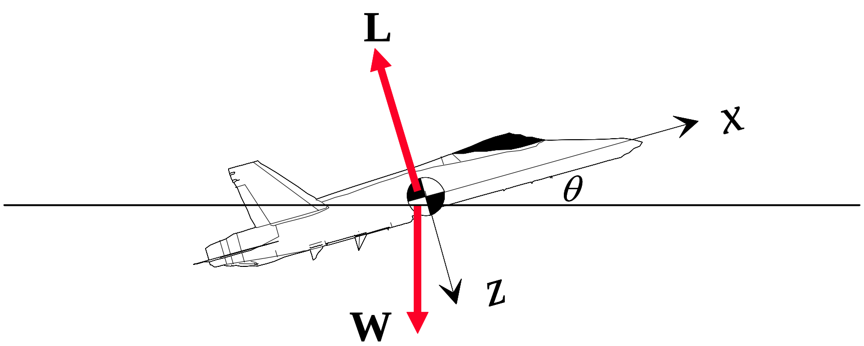
3. Two inelastic masses of 16 and 4 grams move in opposite directions with velocities of 30 and 50 cm/sec respectively. Determine the resultant velocity after impact if they stick together. (V = 14cm/sec)

4. An 8lb body is acted on by a force for a period of 4 sec during which it gains a velocity of 20ft/sec. Determine the magnitude of the force. (1.2lb)

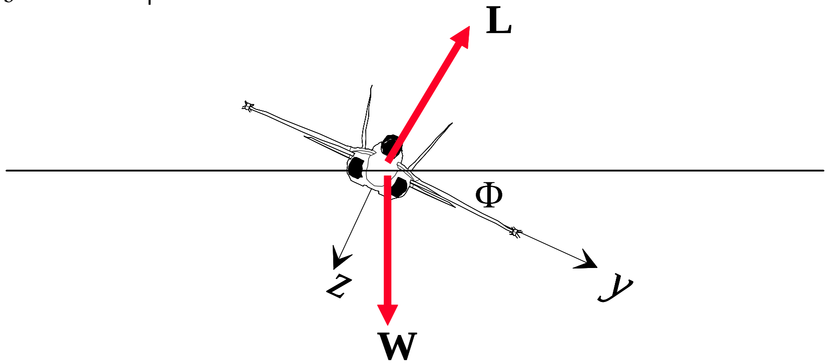
5. A 10-ton locomotive moving at 2ft/sec collides with and is coupled to a 40-ton car at rest on the same straight track. What is their common velocity after impact? (0.4ft/sec)

Axis Transform
Tutorial Solutions

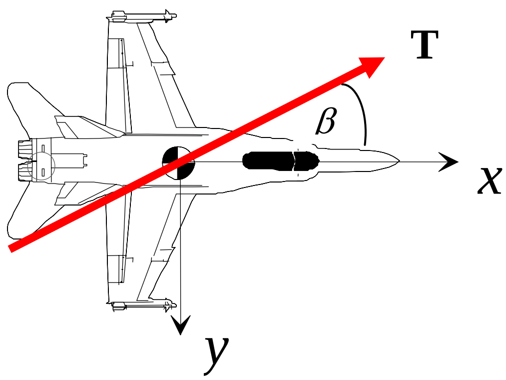
1. Given the following diagram, resolve **W** onto the body axis and determine the following equations
   a. $x_b = -W\sin\theta$
   b. $z_b = W\cos\theta$



Given the following, resolve **W** onto the body axis and determine the following equations
   a. $y_b = W\sin\phi$
   b. $z_b = W\cos\phi$



2. Transform the thrust onto the body axis and determine the following equations
   x_b = Tcosβ
   y_b = -Tsinβ



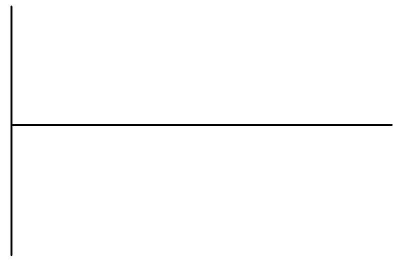
3. Write the following in matrix form

$$x_a = x_b \cos\theta - z_b \sin\theta$$
$$y_a = y_b$$
$$z_a = x_b \sin\theta + y_b \cos\theta$$

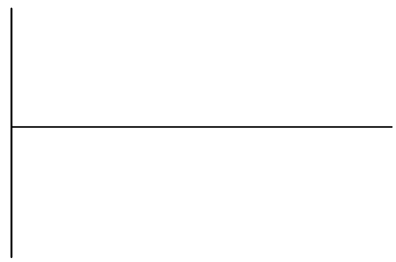
$$\begin{bmatrix} x_a \\ y_a \\ z_a \end{bmatrix} = \begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & \cos\theta & 0 \end{bmatrix} \begin{bmatrix} x_b \\ y_b \\ z_b \end{bmatrix}$$

Motion Analysis
Tutorial Solutions

1. Draw a typical trace for the following <u>oscillating</u> system.
   a. Positive damped (stable)

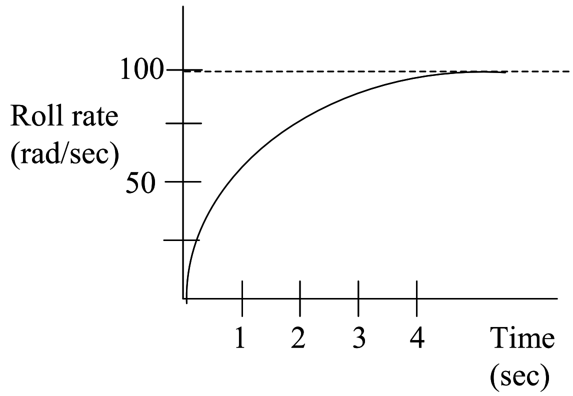


   b. Neutral damped (neutral)



   c. Negative damped (unstable)

2. Given the following 1st order response



   a. Estimate τ = 1 sec

   b. Write the time history response equation x(t) = 100(1- e^{-t})

   c. Is the response convergent or divergent? convergent

3. Given the following "s-domain" equations

   (1) s + .0095 = 0
   (2) $s^2 + .875s + 18.4 = 0$

   a. Find time constant (τ) $s + \frac{1}{\tau} \Rightarrow 0.0095 = \frac{1}{\tau} \rightarrow \tau = 105\sec$

   b. Find natural frequency (ω_n) $\omega^2 = 18.4 \rightarrow \omega = 4.29$

   c. Find damping ratio (ξ) $2\zeta\omega = .875 \rightarrow \zeta = .10$

4. Given the attached trace, calculate the damping ratio (ξ) using the Transient Peak Method.

Average TPR = .59
Thus damping ration = 0.17

5. Given the following, calculate the time constant (τ) using $\tau = \dfrac{\Delta t}{\ln\left(\dfrac{A_1}{A_2}\right)}$

$$\tau = \frac{1}{\ln\left(\frac{30}{20}\right)} = 2.4$$